Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY

Summer Examination-2019

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT3 **Branch: B. Tech (All)**

Semester : 2 **Date : 20/04/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The series $1-\frac{1}{2}+\frac{1}{2^2}-\frac{1}{2^3}+\frac{1}{2^4}-.....\infty$ is

(A) convergent (B) divergent (C) finitely oscillating
(D) infinitely oscillating

b) The interval of convergence of the logarithmic series

$log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots\infty$ is

(A) $-1<x\leq 1$ (B) $-1<x<2$ (C) $-\infty<x<\infty$ (D) $-1\leq x\leq 1$

c) The value of $\int_{-1}^{1}\sin^{11}x\,dx$

(A) 10! (B) $\frac{10}{11}\cdot\frac{8}{9}\cdot\frac{6}{7}\cdot\frac{4}{5}\cdot\frac{2}{5}\cdot\frac{\pi}{2}$ (C) 0 (D) none of these

d) If $f_n=\int_0^{\pi/4}tan^n x\,dx$, then (f_n+f_{n-2}) is equal to?

(A) $\frac{1}{n}$ (B) $\frac{1}{n-1}$ (C) $\frac{n}{n-1}$ (D) $\frac{n-1}{n}$

e) $\int_1^{\infty}\frac{1}{x^{\sqrt{2}}}\,dx$ is convergent.

(A) True (B) False

f) $\left\lceil\frac{1}{2}\right\rceil\left\lceil\frac{3}{2}\right\rceil\left\lceil\frac{5}{2}\right\rceil=$ _____

(A) $\frac{3}{8}(\pi)^{\frac{3}{2}}$ (B) $\frac{3}{8}(\pi)^{\frac{5}{2}}$ (C) $\frac{3}{8}(\pi)^{\frac{1}{2}}$ (D) $\frac{1}{8}(\pi)^{\frac{3}{2}}$

g) $B(1,\ 1)=$ _____

(A) 1 (B) 0 (C) 1/2 (D) none of these

h) The tangents at the origin are obtained by equating to zero

(A) the lowest degree terms (B) the highest degree terms

(C) constant term (D) none of these

i) $\int_0^a \int_0^{\sqrt{a^2-y^2}} dx\, dy$ is equal to

(A) πa^2 (B) $\frac{\pi a^2}{2}$ (C) $\frac{\pi a^2}{4}$ (D) none of these

j) $\int_0^{\frac{\pi}{2}} \int_0^{\infty} e^{-r^2} \cdot r\, dr\, d\theta$ is equal to

(A) $\frac{\pi}{2}$ (B) π (C) $\frac{\pi}{4}$ (D) $-\frac{\pi}{4}$

k) The transformations $x+y=u,\ x-y=v$ transform the area element $dy\, dx$ into $|J|du\, dv$, where $|J|$ is equal to

(A) $\frac{1}{2}$ (B) 1 (C) u (D) none of these

l) The degree and order of the differential equation of all parabolas whose axis is x-axis are

(A) 2, 1 (B) 1, 2 (C) 3, 2 (D) none of these

m) The solution of the equation $x\frac{dy}{dx} = y + x\tan\left(\frac{y}{x}\right)$ is

(A) $\sin\left(\frac{x}{y}\right) = cx$ (B) $\sin\left(\frac{y}{x}\right) = cx$ (C) $\sin\left(\frac{x}{y}\right) = cy$ (D) $\sin\left(\frac{y}{x}\right) = cy$

n) The differential equation of all non-vertical lines in a plane is

(A) $\frac{d^2y}{dx^2} = 0$ (B) $\frac{dy}{dx} = 0$ (C) $\frac{dx}{dy} = 0$ (D) $\frac{d^2x}{dy^2} = 0$

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Test the convergence of the series $\frac{1}{\sqrt{2}-1} + \frac{1}{\sqrt{3}-1} + \frac{1}{\sqrt{4}-1} + \ldots\ldots$ **(5)**

b) Using reduction formula evaluate: $\int_0^{\pi} x\sin^7 x\cos^4 x\, dx$ **(5)**

c) Prove that $\int_0^{\infty} \frac{x^4}{4^x} dx = \frac{24}{(\log 4)^5}$. **(4)**

Q-3 **Attempt all questions** **(14)**

a) Prove that $\int_0^1 x^5\left(1-x^3\right)^{10} dx = \frac{1}{3}B(2,\ 11)$. **(5)**

b) Using reduction formula prove that $\int_0^{\pi} x\cos^6 x\, dx = \frac{5\pi^2}{32}$. **(5)**

c) Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^2}$. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Change the order of integration in the integral $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\, dy\, dx$ and hence evaluate it. **(5)**

b) Examine the series $\sum_{n=1}^{\infty} \frac{x^n}{n^p}$ for convergence using root test. **(5)**

c) Solve: $\frac{(x-2y)}{(3x+y)}\frac{dy}{dx} = 3x^2 - 5xy - 2y^2$ **(4)**

Q-5 **Attempt all questions** **(14)**

a) Solve: $\left(xy^2 + e^{-\frac{1}{x^3}}\right)dx - x^2 y\, dy = 0$ **(5)**

b) By changing the transformations $x+y=u,\ y=uv$, show that $\int_0^1 \int_0^{1-x} e^{\frac{y}{(x+y)}}\, dy\, dx = \frac{e-1}{2}$. **(5)**

c) Using reduction formula evaluate: $\int_0^1 \frac{x^6}{(1+x^2)}\, dx$ **(4)**

Q-6 **Attempt all questions** **(14)**

a) Prove that $\int_0^{\frac{\pi}{2}} \frac{dx}{\tan^p x} = \frac{\pi}{2}\sec\left(\frac{p\pi}{2}\right)$. **(5)**

b) Solve: $\left(x^2 + y^2 + 1\right)dx - 2xy\, dy = 0$ **(5)**

c) Evaluate: $\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} \left(x^2 + y^2 + z^2\right) dz\, dy\, dx$ **(4)**

Q-7 **Attempt all questions** **(14)**

a) Trace the curve $xy^2 = 4a^2(2a - x)$. **(5)**

b) Show that $\int_1^{\infty} \frac{\ln x}{x^2}\, dx$ converges and compute its value. **(5)**

c) Find the area enclosed by the cardioid $r = a(1 - \cos\theta)$. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Find the volume of the solid generated by the revolution of the loop of the curve $x(x^2 + y^2) = a(x^2 - y^2)$. **(5)**

b) Find the asymptotes of the curve $y^3 - x^2(6 - x) = 0$. **(5)**

c) Evaluate: $\int_0^{\infty} \frac{dv}{(1+v^2)(1+\tan^{-1} v)}$ **(4)**